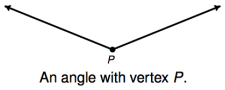
MATH 8 UNIT 1- Introduction to Angles

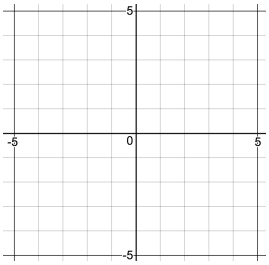
This material can be found in section 5.1 and 6.1, but is covered more in depth here.

Angles and Degree measure

Terminology: Initial side, terminal side, standard position, positive/negative direction, complementary, supplementary, right angle, quadrantal angle, coterminal, greek names, etc.

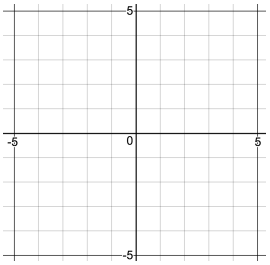


An angle with vertex *P*.

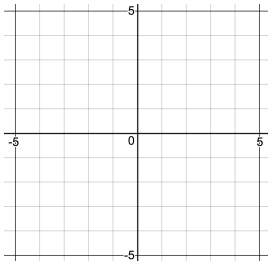


Degree Measure: 1 rotation = _____

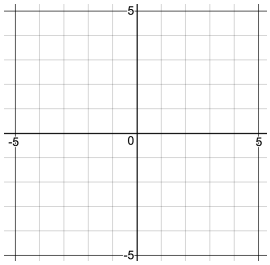
Breaking up one revolution to get a feel for the relative size of key angles:



Quadrantal Angles
(1/4 of revolution)



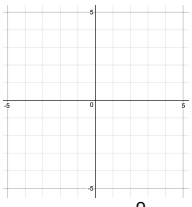
45 degrees
(1/8 revolution)
Divide quadrant in half

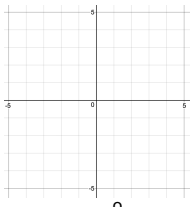


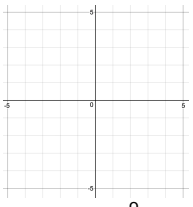
30, 60 degrees
(1/12 , 2/12 of revolution)
Divide quadrant in thirds

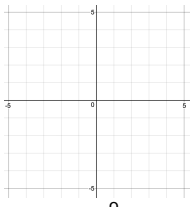
Locating Angles – Reference Angles

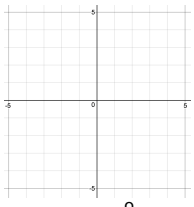
Make a rough sketch of each of the following angles in standard position. HOW did you decide where they were located?


160°


250°


400°


-178°

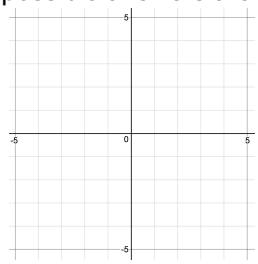

- 260°

Reference angles can help us determine where an angle is located. A reference angle is the **acute** angle formed by the terminal side of a given angle, θ, and the nearest portion of the x-axis. Find the reference angles for each of the angles above.

Using Reference Angles

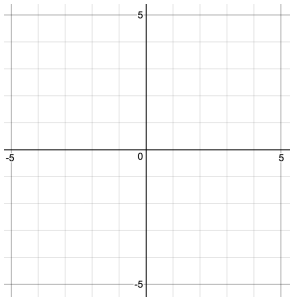
Going "backwards", sometimes we are given a reference angle and a quadrant corresponding to the terminal side of the angle and asked to locate the angle.

EX: Sketch an angle with reference angle of 10 degrees whose terminal side is in Quadrant 3. What is the measure of this angle? Give an angle coterminal with this angle. How many possible answers are there?

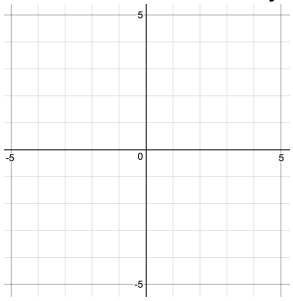


Other times the quadrant is not specified.

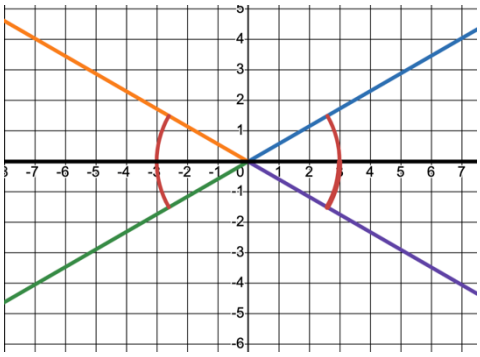
EX: Sketch terminal sides of all angles having reference angle of 30 degrees. How many such terminal sides are there? How many possible angles? What are their measure?



EX: Sketch terminal sides of all angles having reference angle of 83 degrees. Again, there are 4 possible terminal sides, infinitely many angles. What are their measure?



Generalizing, the following terminal sides all have a reference angle of θ



do Angle worksheet 1 (degrees))

Converting Decimal Degrees $\Leftrightarrow$ Degree-Minute-Second (DMS) :

First, simple conversions using “dimensional analysis”

The conversion factor is 1 foot = 12 inches

Convert 80 inches to feet.

Convert 5 feet to inches.

When work with angles, we sometimes give angles in decimal degrees form, other times we use the DMS convention (degrees-minutes-seconds)

The conversion factors are: 1 degree = 60 minutes , $1^\circ = 60'$
1 minute = 60 seconds, $1' = 60''$

Example:
DMS -> Decimal Degrees

Decimal Degrees→ DMS

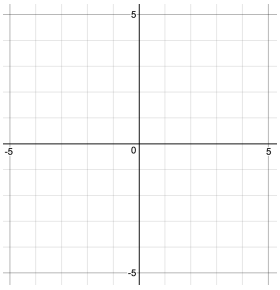
Angles and Radian Measure

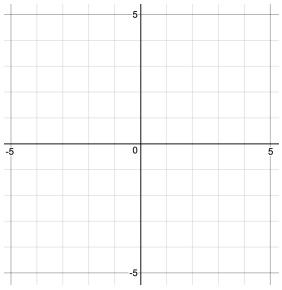
Now consider a new way to measure angles, called radian measure. For now, we will approach radian using the same logic as we did with degrees

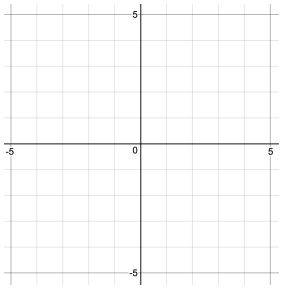
For now, let's suppose 1 revolution = _____ radians.

(In the next unit we will look more deeply at radians and better understand where this number comes from).

As we did with degrees, we break up one revolution to get a feel for the relative size of key angles:







Quadrantal Angles

(1/4 of revolution)

_____

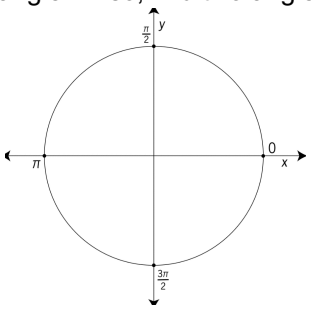
(1/8 revolution)
Divide quadrant in half

_____

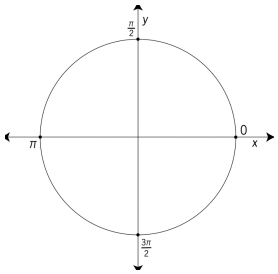
(1/12 , 2/12 of revolution)
Divide quadrant in thirds

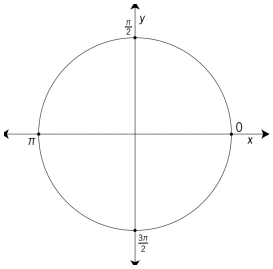
Locating angles in Radians – Same logic but more arithmetic

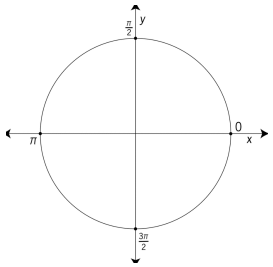
Locate $\theta = 5\pi/6$ by comparing it to quadrantal angles to determine quadrant. Find and use reference angle. Also, find two angles coterminal with $\theta = 5\pi/6$

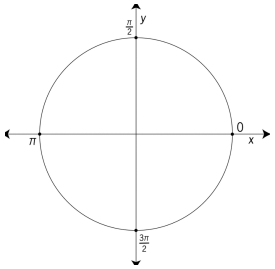


Locate each of the following angles(radians understood unless degrees specified) and find ref. angle.





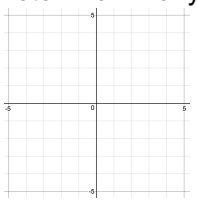




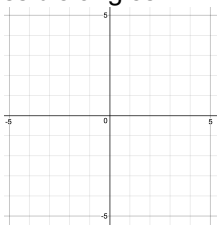
$\theta = 5\pi / 3$ $\theta = 3\pi / 4$ $\theta = 21\pi / 12$ $\theta = -7\pi / 8$

And backwards, given a reference angle, find the following described angle:

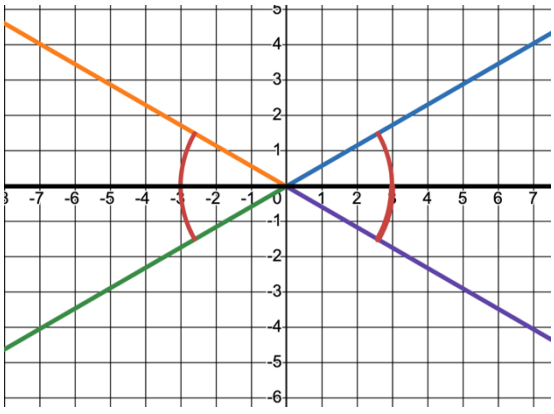
EX: Sketch an angle with reference angle of $\theta = \pi/9$ whose terminal side is in Quadrant 2.
Note: How many possible answers are there? Give an angle coterminal with this angle.



Other times the quadrant is not specified.
EX: Sketch angles with reference angle of $\theta = \pi/3$. How many such terminal sides are there? How many possible angles?



Generalizig: Sketch terminal sides of all angles having reference angle of θ radians. Again, there are 4 possible terminal sides, infinitely many angles. What are their measure?



Do Angle worksheet 2 (radians), then do Angle WS 3 (degrees and radians)

Important Advice: Practice radians so you are comfortable and can "think in radians".

Converting Degrees-Radians: 1 revolution = $2\pi \text{ radians} = 360^\circ$, so conversion factor is $\pi \text{ radians} = 180^\circ$

1) Convert to radians

a) 30° b) 45° c) 60° d) 80°

2) Convert to degrees

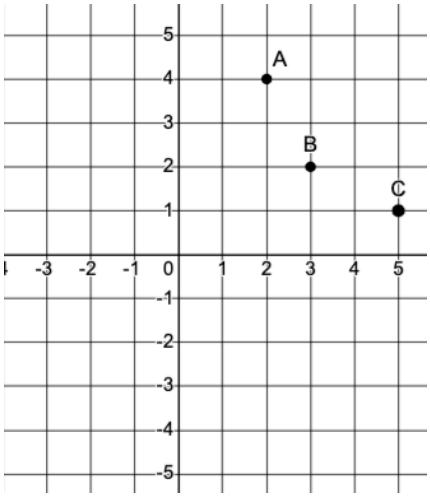
a) $\frac{\pi}{10}$ b) $\frac{5\pi}{12}$ c) 1

Points on the Terminal Sides of Angles

Many times, we are interested in points known to be on terminal sides of angles.

Warm up Questions:

Given the points shown, answer the following questions.



Coordinates of point which is symmetric to A relative to the y axis____

Coordinates of point which is symmetric to A relative to the x axis___

Coordinates of point which is symmetric to A relative to the origin.___

List the points in ascending order according to their y value. _____

List the points in ascending order according to their x value. _____

In the given diagram, the points have coordinates (in no particular order) $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$, $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$, $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

Decide which one is A, B, and C and use this information and your knowledge of symmetry to find the following: (try not to draw the full circle with all the angles and points, it will help you in the long run)